Grand Unification

strong + weak

electromagnetic

interactions

problems

three unrelated coupling constants?

---

quantization of electric charges?

=>

Q(neutrino) = 0

Q(electron) = -1

Q(u) = 2/3

Q(d) = -1/3

SU(3) x SU(2) x U(1)

semisimple

group

(Pati / Salam - 1974)

leptons as a fourth color

SU(3) x SU(2) x U(1)

< SU (4) x SU (2, L) x SU (2, R)

$$\begin{bmatrix} \nu_e & \vdots & u & u & u \\ e^- & \vdots & d & d & d \end{bmatrix}$$

SU(4)

$$\begin{pmatrix} \nu_e & \vdots & u & u & u \\ e^- & \vdots & d & d & d \end{pmatrix}_L = (4,2,1)$$

$$\begin{pmatrix} \nu_e & \vdots & u & u & u \\ e^- & \vdots & d & d & d \end{pmatrix}_R = (4,1,2)$$

righthanded analogue of SM neutrino

$$\begin{bmatrix} \nu_e & \vdots & u & u & u \\ e^- & \vdots & d & d & d \end{bmatrix}_R = (4,1,2)$$

new "righthanded„ W-bosons

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \qquad (e^-_R)$$

$$\Downarrow$$

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \qquad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_R$$

$$M_{W_L} \ll M_{W_R}$$

SU(4) ==> SU(3) x (B-L)

$$Q_{EM} = T_{3L} + T_{3R} + \frac{1}{2}(B - L)$$

$$B : baryon - number$$

$$L : lepton - number$$

Grand Unification

SU(3) x SU(2) x U(1)

subgroup

of

simple group

SU(3) x SU(2) x U(1)

⊂ SU(5)

Georgi – Glashow

1974

SU(3) x SU(2) x U(1)

$\subset$ SO(10)

Fritzsch – Minkowski

Georgi

1975

rank of a group:
number of diagonal elements

SU(2):

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

rank of SU(2) ➔ 1

rank of a group: number of diagonal elements

rank U(1) = 1

rank SU(n)=n-1

rank SU(2)=1 rank SU(3)=2

rank SU(3) x SU(2) x U(1) =4

rank SU(5) = 4

===>

SU(5)

smallest group

SU(5) - theory

SU(3) x SU(2) x U(1) < SU(5)

fermions of one family in

two different representations

$$(10) + (\bar{5})$$

5-represenation:

$$5 \to (3,1) + (1,2)$$

$$\overline{5} \to (\overline{3},1) + (1,2)$$

10-represenation:

$$10 \to (\overline{3},1) + (3,2) + (1,1)$$

(antisymmetric second rank tensor)

$$\left(\overline{5}\right) = \begin{pmatrix} \overline{d}_r \\ \overline{d}_g \\ \overline{d}_b \\ \nu \\ -e^- \end{pmatrix}$$

sum of electric charges = 0

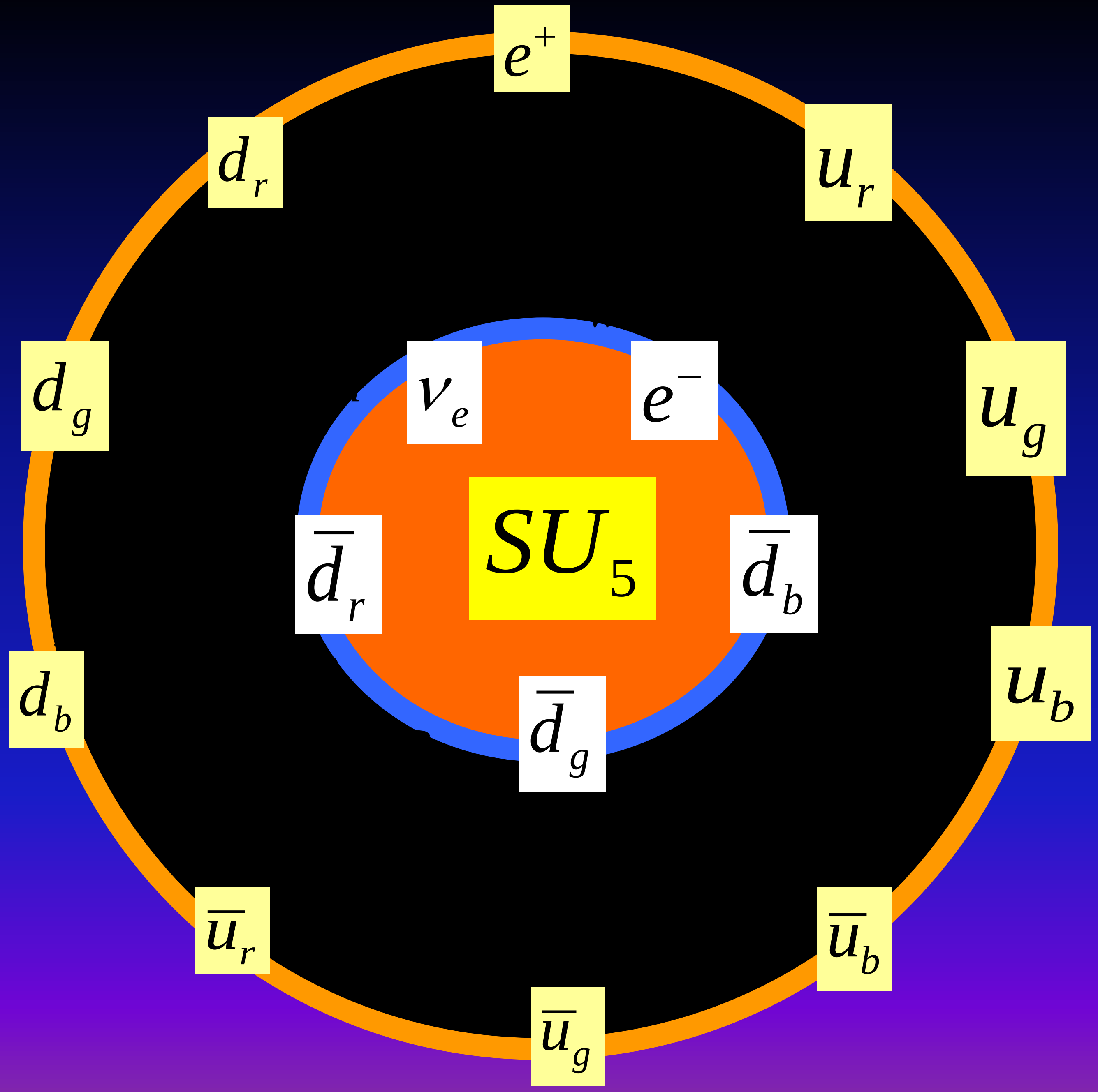
=> charge quantization

$$
(10) = \begin{pmatrix} 0 & \overline{u}_b & -\overline{u}_g & u_r & d_r \\ & 0 & \overline{u}_r & u_g & d_g \\ & & 0 & u_b & d_b \\ & & & 0 & \overline{e} \\ & & & & 0 \end{pmatrix}
$$

sum of electric charges = 0

quarks and leptons in the same multiplet

=> decay of the proton into leptons ???
(later)


e^+
d_r
u_r
d_g
u_g
ν_e
e^-
SU_5
$\bar{d}_r$
$\bar{d}_b$
$\bar{d}_g$
d_b
u_b
$\bar{u}_r$
$\bar{u}_b$
$\bar{u}_g$

lepton – quark family

$$\bar{5} + 10 \rightarrow (3,2) + 2(\bar{3},1) + (1,2) + (1,1)$$

SU(5) ==> SU(3) x SU(2) x U(1)

$$\bar{5} + 10 \rightarrow (3,2) + 2(\bar{3},1) + (1,2) + (1,1)$$

u d

$\bar{u}$ $\bar{d}$

neutrino electron

positron

Gauge bosons in SU(5) > SU(3) x SU(2) x U(1):

adjoint representation ~ (24)

5 x 5* = 24 + 1

24 = (8,1) + (1,3) + (1,1) + (3,2) + (3*,2)

24 =

12 gauge bosons:
8 gluons, 2 W-bosons,
one Z boson, one photon

+

12 superheavy color
triplet gauge bosons

gluons

$W^+ - W^- - Z - \gamma$

24 = (8,1) + (1,3) + (1,1)

+

(3,2) + (3*,2)

superheavy gauge bosons

Interesting features of SU(5):

1. electric charge => quantized

(charge operator is generator of a simple group)

2. Electric charges of quarks are related to charge of electron

$$trQ = 0$$

$$\rightarrow Q_d = \frac{1}{3} Q_{e^-}$$

Grand unification takes place above a very high unified mass scale M. There is only one unified coupling constant g. The coupling constants for QCD and for the SU(2) x U(1) theory are the same, apart from Clebsch-Gordan coefficients.

The different values of the three coupling constants at low energies are due to renormalization effects.

$$SU(5)$$

$$\Downarrow$$

$$SU(3)\otimes SU(2)\otimes U(1)$$

$$\Downarrow$$

$$SU(3)\otimes U(1)$$

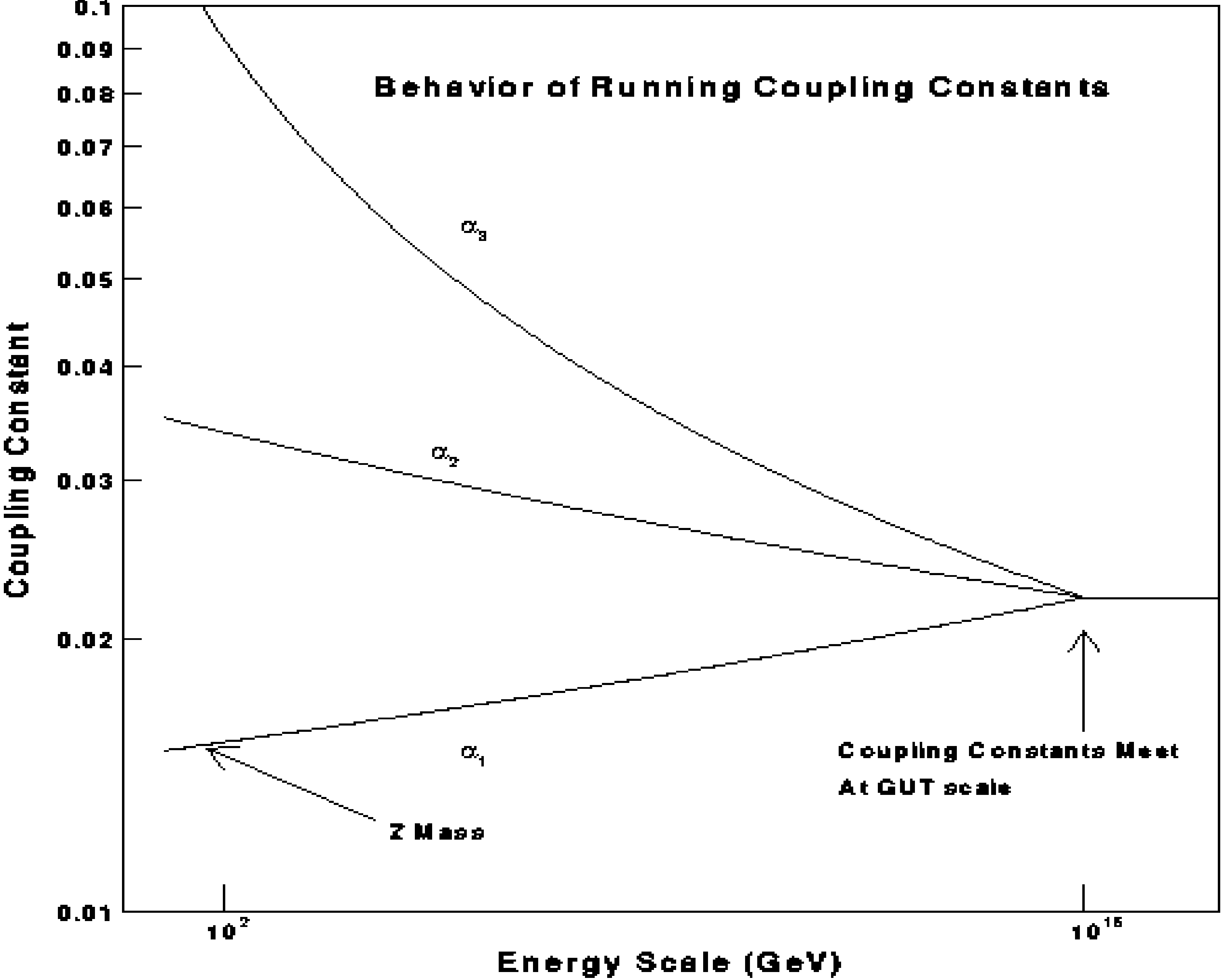
$$\alpha_{GUT}$$

$$\Downarrow$$

$$\alpha_s \otimes \alpha_2 \otimes \alpha_1$$

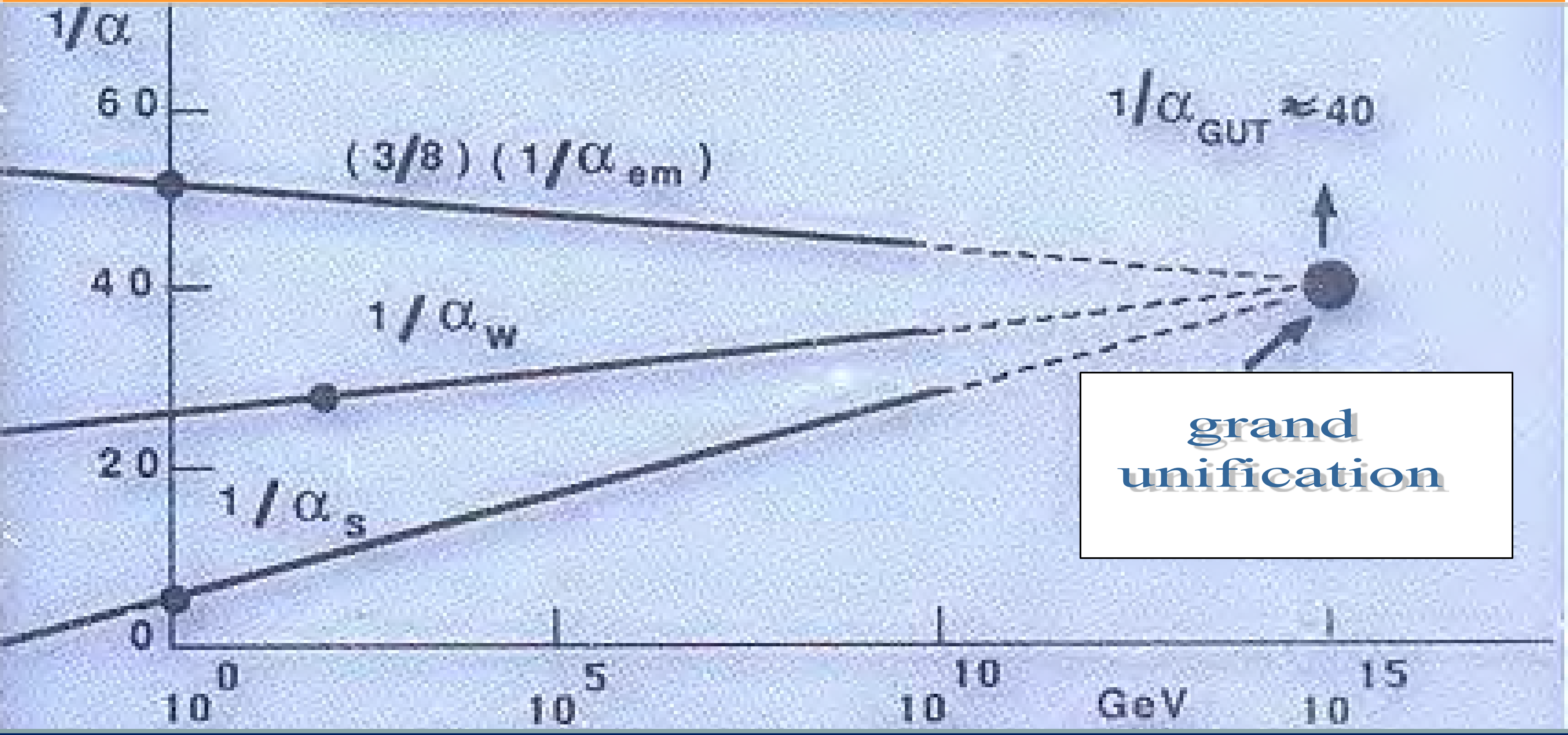
$$\Downarrow$$

$$\alpha_s \otimes \alpha$$



Behavior of Running Coupling Constants
Coupling Constant
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
α_3
α_2
α_1
Z Mass
Coupling Constants Meet
At GUT scale
10^2
10^{15}
Energy Scale (GeV)

convergence of the coupling constants



SU(5)

SU(3) x SU(2) x U(1)

SU(3) x U(1)

The convergence of the three coupling constants at M implies a condition at low energies:

$$\frac{g_1}{g_2} = \tan\theta$$

$$\rightarrow \sin^2\theta = \frac{trT_3^{\ 2}}{trQ^2}$$

$$\rightarrow \sin^2\theta = \frac{trT_3^{\ 2}}{trQ^2}$$

This formula is independent of the details of the grand unified theory, e.g. the gauge group.

It depends on the spectrum of the fermions, described in the Standard Model.

$$\rightarrow \sin^2\theta = \frac{trT_3^{\ 2}}{trQ^2}$$

$$\sin^2\theta = \frac{2\cdot(1/2)^2(3+1)}{\left\{\left[(2/3)^2+(1/3)^2\right]3+1\right\}^2} = \frac{3}{8}$$

$$\frac{4\pi}{g_3{}^2(\mu)} = \frac{1}{\alpha_s(\mu)} = \frac{1}{\alpha_{GUT}} + \frac{1}{6\pi}(4F - 33)\log(M/\mu)$$

$$\frac{4\pi}{g_2{}^2(\mu)} = \frac{\sin^2\theta}{\alpha(\mu)} = \frac{1}{\alpha_{GUT}} + \frac{1}{6\pi}(4F - 22)\log(M/\mu)$$

$$\frac{4\pi}{g_1{}^2(\mu)} = \frac{3}{5}\frac{\cos^2\theta}{\alpha(\mu)} = \frac{1}{\alpha_{GUT}} + \frac{1}{6\pi}4F\log(M/\mu)$$

F: Nr. of families

==>

$$\frac{\sin^2\theta}{\alpha} - \frac{1}{\alpha_s} = \frac{1}{6\pi} 11 \cdot \log(M / \mu)$$

$$\frac{\alpha}{\alpha_s} = \frac{3}{10}(6\sin^2\theta - 1)$$

$$\frac{1}{\alpha} = \frac{8}{3}\frac{1}{\alpha_{GUT}} + \frac{1}{6\pi}\left(\frac{32}{3}F - 22\right)\log\frac{M}{\mu}$$

At unification energy M:

$$\alpha_{GUT} = \frac{8}{3}\alpha$$

LEP data

No convergence
of coupling constants

in SU(5)

$$\frac{\alpha}{\alpha_s} = \frac{3}{10}(6\sin^2\theta - 1)$$

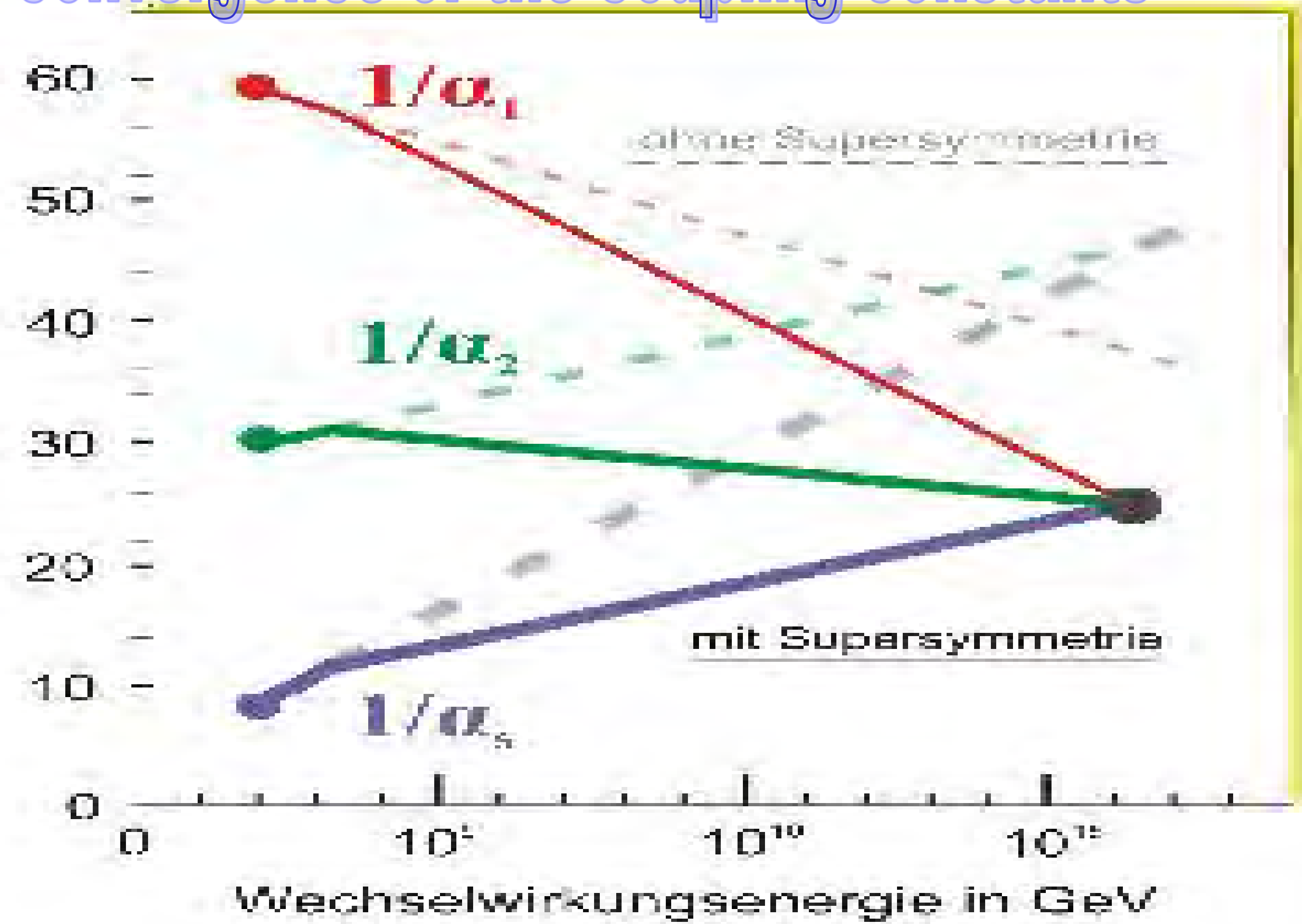
$$\sin^2\theta_W \approx 0.2315$$

$$\alpha \approx 1/128$$

$$==> \alpha_s \approx 0.067$$

experiment: 0.118

convergence of the coupling constants



convergence in case of supersymmetry

energy scale of supersymmetric partners:

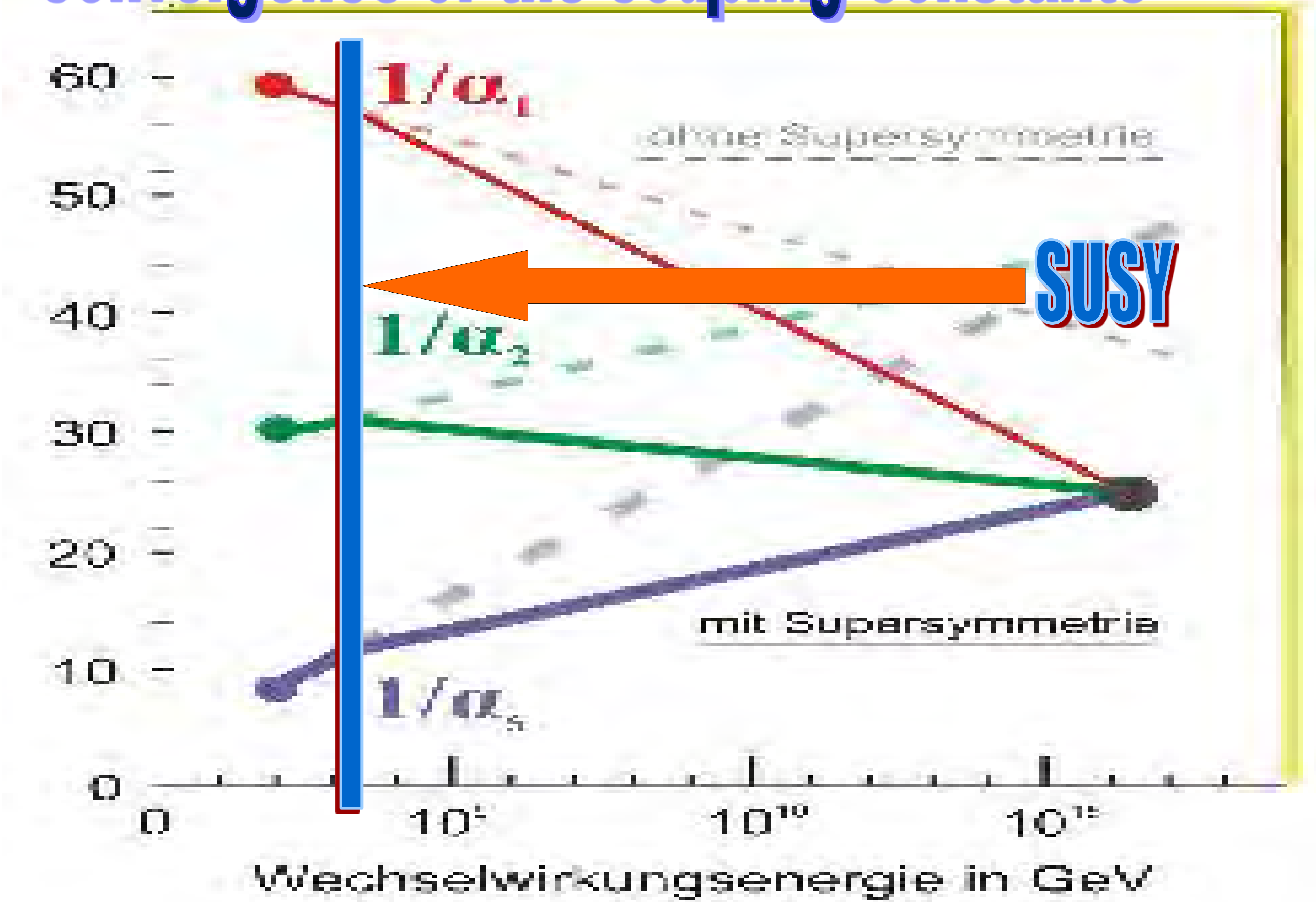
~ 1 TeV

each particle of the Standard Model

➔supersymmetric particle

quark	***squark*** (spin 0)
lepton	***slepton*** (spin 0)
photon	***photino*** (spin ½)
gluon	***gluino*** (spin ½)
W – boson	***wino*** (spin ½)

convergence of the coupling constants



Grand unification

SU(3) x SU(2) x U(1)

==>SO(10)

Fritzsch - Minkowski

Georgi - 1975

n complex numbers can be described by

(2n) real numbers.

SU(5) ==> SO(10)

$$SU(3) \times SU(2) \times U(1)$$

$$< SU(4) \times SU(2,L) \times SU(2,R)$$

$$\sim SO(6) \quad \times \quad SO(4)$$

$$\Rightarrow SO(10)$$

$$SO(10):$$

$$real \quad 10x10 \quad matrices \quad O$$

$$O^T O = 1 \qquad \det O = 1$$

representations
SO(10)

16-dimensional

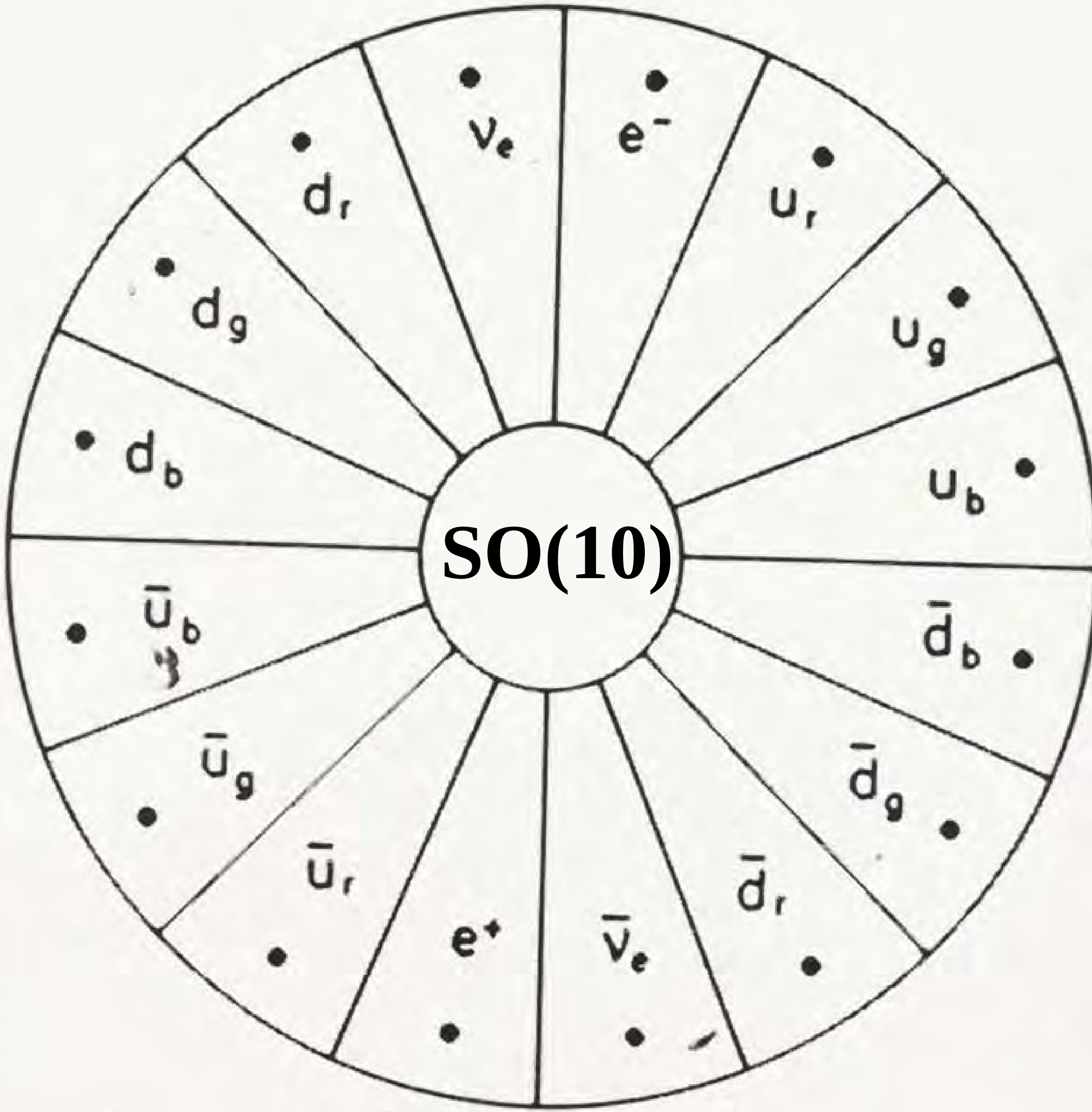
spinor

representation

SO(10) → 16

SO(3) → *Pauli spinor*

16:


SO(10)
ν_e
e^-
u_r
u_g
u_b
$\bar{d}_b$
$\bar{d}_g$
$\bar{d}_r$
$\bar{\nu}_e$
e^+
$\bar{u}_r$
$\bar{u}_g$
$\bar{u}_b$
d_b
d_g
d_r

$$16 = \begin{bmatrix} \nu_e & : u \quad u \quad u & : \bar{u} \quad \bar{u} \quad \bar{u} & : \bar{\nu}_e \\ e^- & : d \quad d \quad d & : \bar{d} \quad \bar{d} \quad \bar{d} & : e^+ \end{bmatrix}$$

Adjoint representation:

SO(10) : 45

SU(2): 3

SU(3): 8

SU(5): 24

Breaking of SO(10):

2 different breaking patterns

SO(10)

↓

SU(5) x U(1)

SO(10) ==> SU(5) x U(1)

$$16 = \bar{5} + 10 + 1$$

(1: righthanded neutrino !)

1: can have a large Majorana mass

neutrinos would then be massive,
mass generation via the see-saw mechanicsm

SO(10) ==> SU(5) x U(1)

gauge coupling unification
like in SU(5) theory
(problem with convergence)

Breaking of SO(10)

2 different breaking patterns

SO(10)

↓

SU(4) x SU(2,L) x SU(2,R)

$$SO(10) \Longrightarrow SU(4)_c \times SU(2)_L \times SU(2)_R$$

2 new energy scales:

breaking of SU(4)
and SU(2,R)

M(EW) => M(R) => M(4) => M(GUT)

SO(10)

SO(6) x SO(4)

SU(4) x SU(2,L) x SU(2,R)

SU(3) x SU(2,L) x U(1)

SU(5): two steps

SO(10): three steps

SO(10)

two intermediate energy scales

.. ➔

no problem with convergence

of coupling constants

SU(4) x SU(2,L) x SU(2,R)

$\downarrow \Lambda_R$ $\qquad$ $\downarrow \Lambda_R$

SU(3) x SU(2,L) x U(1)

Dirac neutrino mass

$$D \cdot \left\{ \left(\overline{\nu}_R \nu_L\right) + \left(\overline{\nu}_L \nu_R\right) \right\}$$

Preferred in SO(10):

Majorana mass

$$M \cdot \left(\nu_R \nu_L\right)$$

See-saw mechanism

$$\left(\overline{\nu_L}, \overline{\nu_R}\right)\begin{pmatrix} 0 & D \\ D & \Lambda_R \end{pmatrix}\begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}$$

neutrino masses:

$$m_1 = D^2 / \Lambda_R$$

$$m_2 \approx \Lambda_R$$

Breaking of SU (4) to SU (3)

Breaking of SU (2,R)

Mass of righthanded Majorana neutrino

at energy Λ_R

Higgs fields for Yukawa couplings to the fermions:

$$16_F \times 16_F = (10) + (120) + (126)$$

SO(10) representations: (10), (120) and (126)

$$16_F \times 16_F = (10) + (120) + (126)$$

$16_F \times 16_F = (10) + (120) + ($[redacted]$)$

==> texture zeros in fermion mass matrices

Grand Unified Theories

beyond

SU(5) - SO(10)

Continuous groups:

SO(n)

SU(n)

Sp(n)

Exceptional groups:

G(2), F(4), E(6), E(7), E(8)

S O (n)

S U (n)

~~S p (n)~~

exceptional groups

G(2) - F(4)

E(6) - E(7)

E(8)

rank of G(2): 2

rank of F(4): 3

===>

cannot be used for grand unification

Grand unification, based on exceptional gauge groups

group E(6)

smallest representations:
(27) - fundamental -,
(78) - adjoint.

fermions
27 representation

gauge bosons

78 representation

E(6) ==> SO(10)

27 ==> 16 + 10 + 1

11 exotic fermions expected

(masses must be very high)

$$E(6) \Longrightarrow SO(10)$$

$$78 \Longrightarrow 45 + 16 + \overline{16} + 1$$

gauge bosons of SO(10)

other breaking:

$$E(6) \Rightarrow SU(3)_C \otimes SU(3)_L \otimes SU(3)_R$$

$$(27) = (3,3,1) + (3^{\otimes},1,3^{\otimes}) + (1,3^{\otimes},3)$$

flavor group $SU(3)_L \otimes SU(3)_R$

Grand unification, based on the group E(8)

E (8):

smallest nontrivial representation = adjoint representation: (248)

➔Fermions and bosons are both in a 248 representation

➔ supersymmetry ?

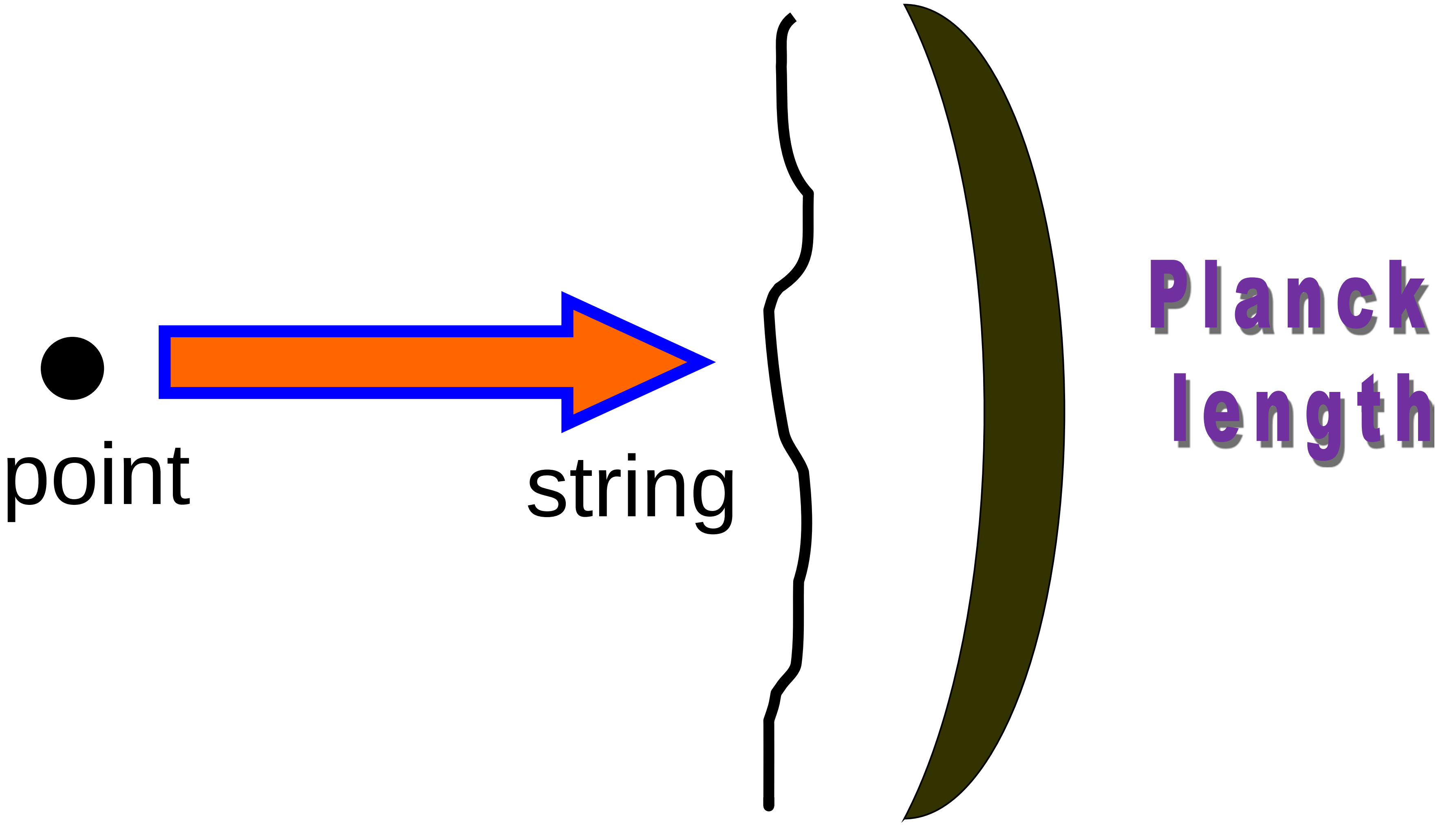
Energy scale of grand unification only 4 orders of magnitude smaller than Planck mass

Planck mass

$$m_P = \sqrt{\frac{\hbar c}{G}}$$

$$\cong 1.22 \bullet 10^{19}\, GeV \approx 2.2 \bullet 10^{-8}\, kg$$

superstring theory



Planck length

$$l_{Planck} = \sqrt{\frac{\hbar G}{c^3}} \cong 1.616 \bullet 10^{-35} m$$

$\approx 10^{-20} \bullet$ radius of proton *example:*

superstrings

↓

E(8)

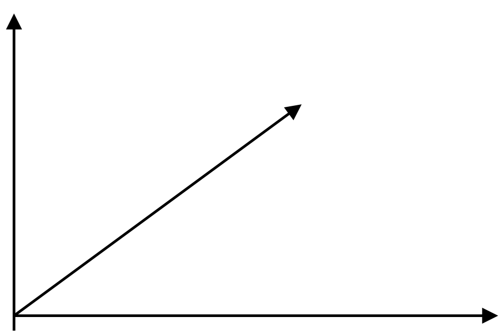
↓

standard model

11 dimensions of space-time

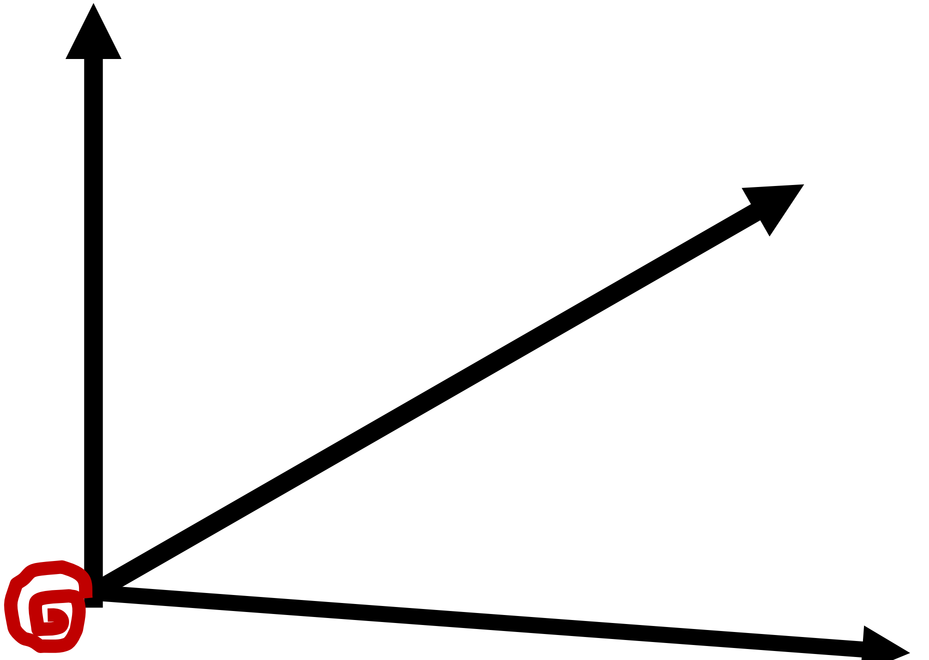
7 space dimensions curled up:

3



4 dimensions of space

1 space dimension curled up



Proton Decay

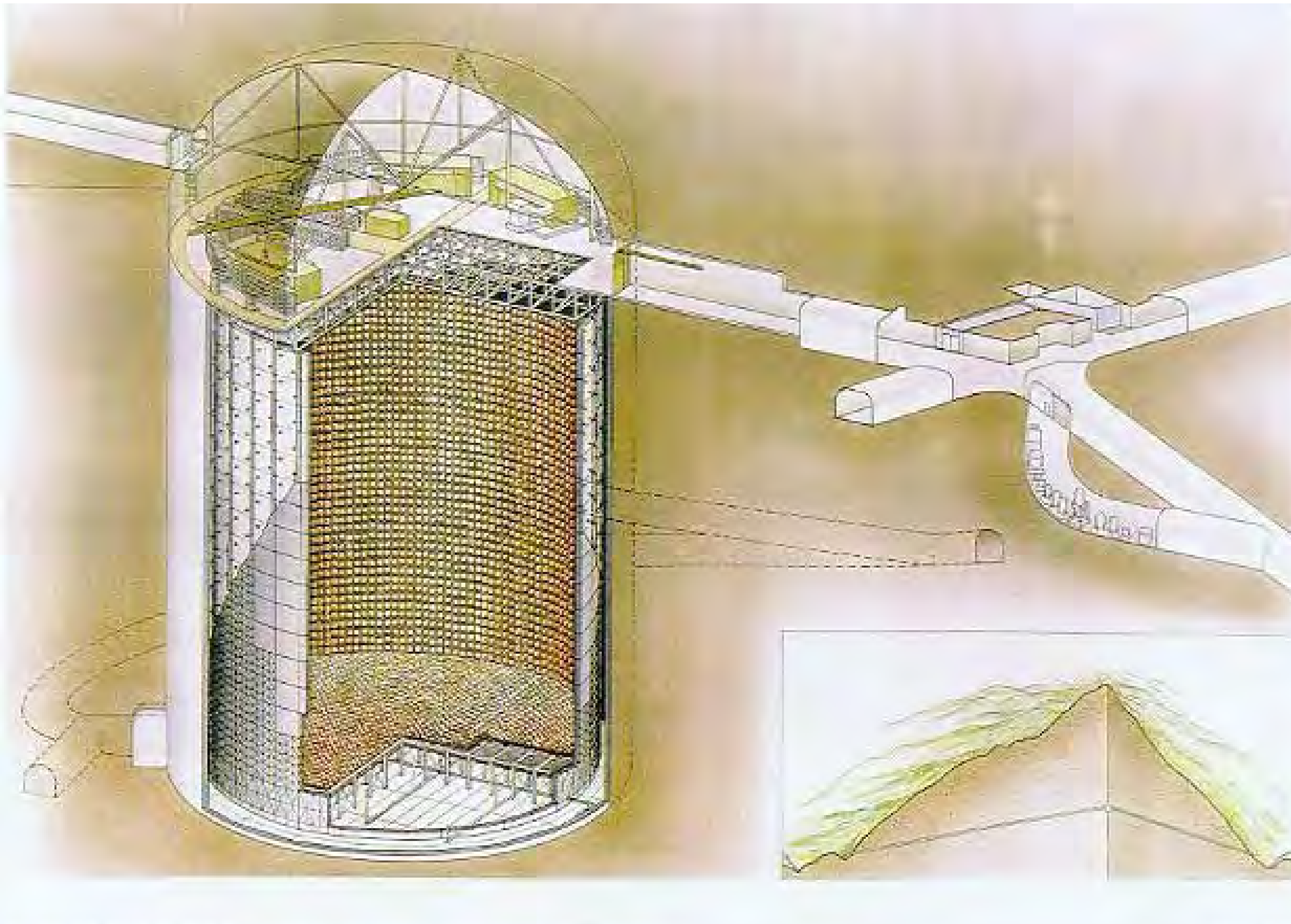
Experiments in Mozumi Mine
near Kamioka south of Toyama
in Japan
(zinc, silver,)

Proton Decay:

limit from experiments (Kamioka):

lifetime > 10^{32} years

Kamioka
KEK
HOKKAIDO
HONSHU
SHIKOKU
Sea of Japan
Pacific Ocean
Pacific Ocean
NANSEI (Southwest)
Kunashiri isl.
Sado isl.
Oki isl.
Notto pen.
Izu pen.
TOKYO
NAGANO
TOYAMA
KANAZAWA
NIIGATA
SENDAI
KYOTO
OSAKA
NAGOYA
HIROSHIMA
SAPPORO



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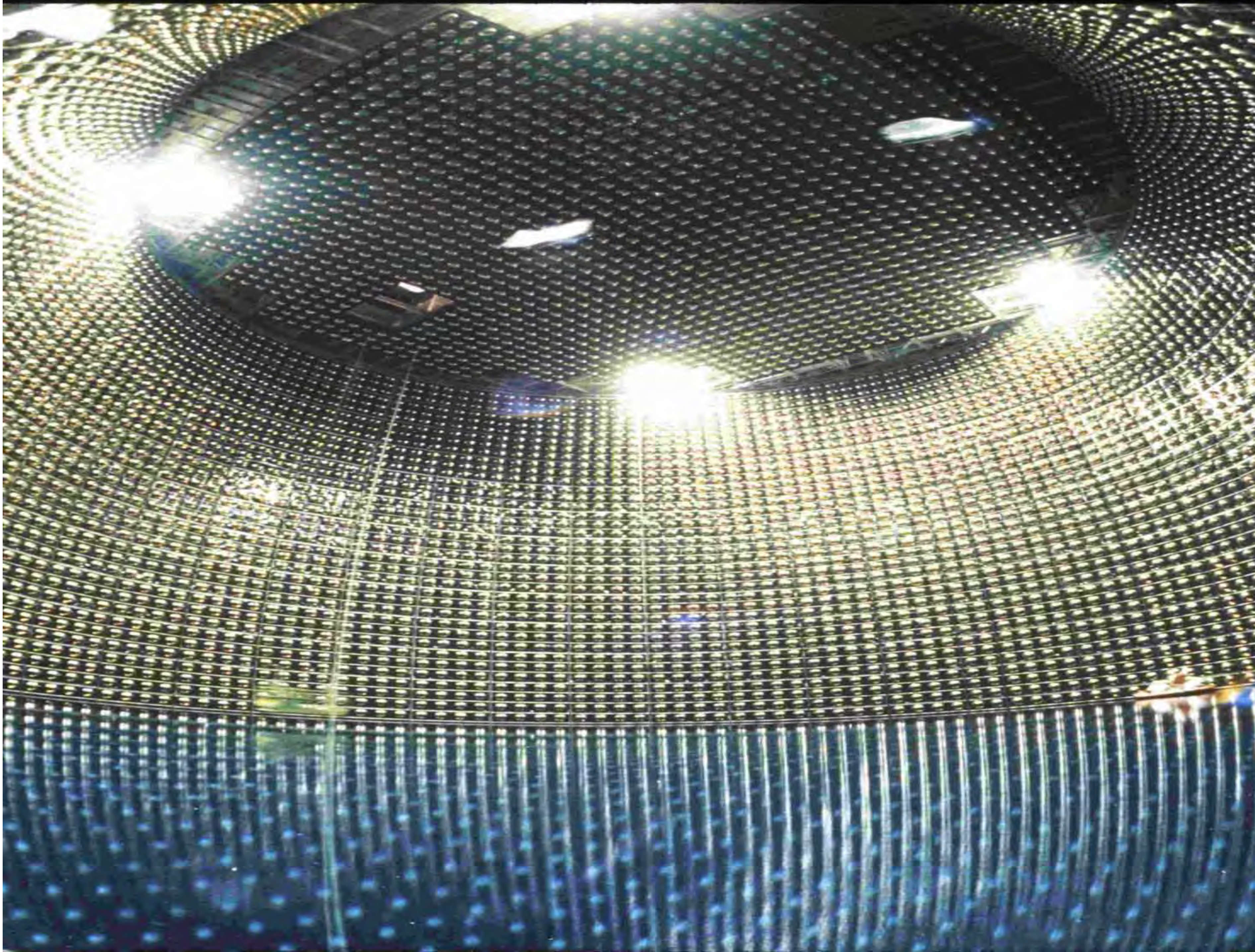


proton decay

in

Grand Unified Theories

Pati - Salam

leptons as fourth color

ν_e	u	u	u
e^-	d	d	d

===>> **proton decay**

possible decays:

$$p \Rightarrow 3\nu + \pi^{+}$$

$or:$

$$p \Rightarrow 4\nu + e^{+}$$

(of 3rd order in gauge coupling, no strong limit)

SU(5) -theory:

$$\overline{5} = \begin{pmatrix} \bar{d}_r \\ \bar{d}_g \\ \bar{d}_b \\ \nu \\ e^- \end{pmatrix}$$

leptons and quarks in the same representation

==> *proton decay*

proton decay mediated by exchange of superheavy gauge bosons

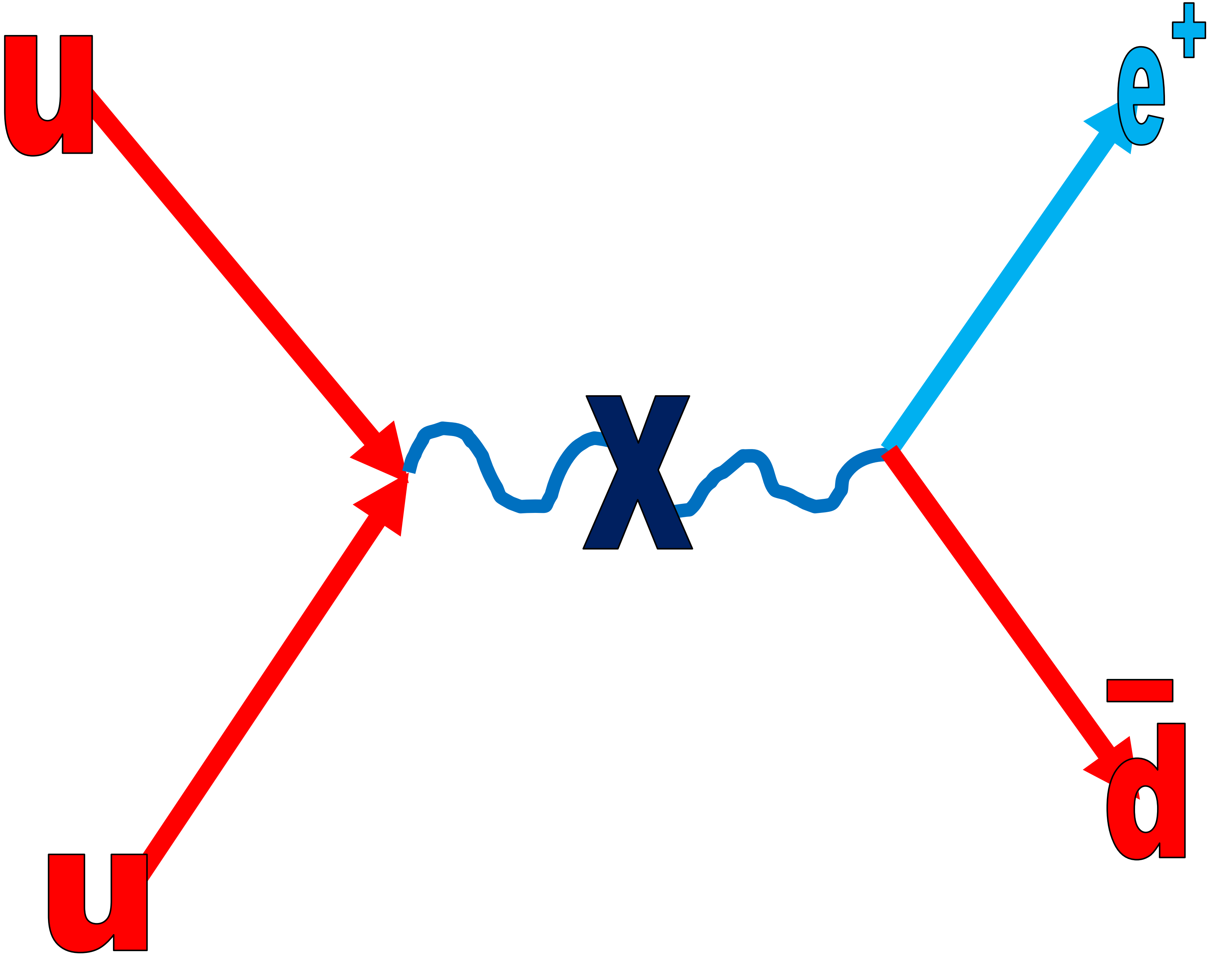
$$24 = (8,1) + (1,3) + (1,1)$$

$$+(3, 2) + (3^*,2)$$

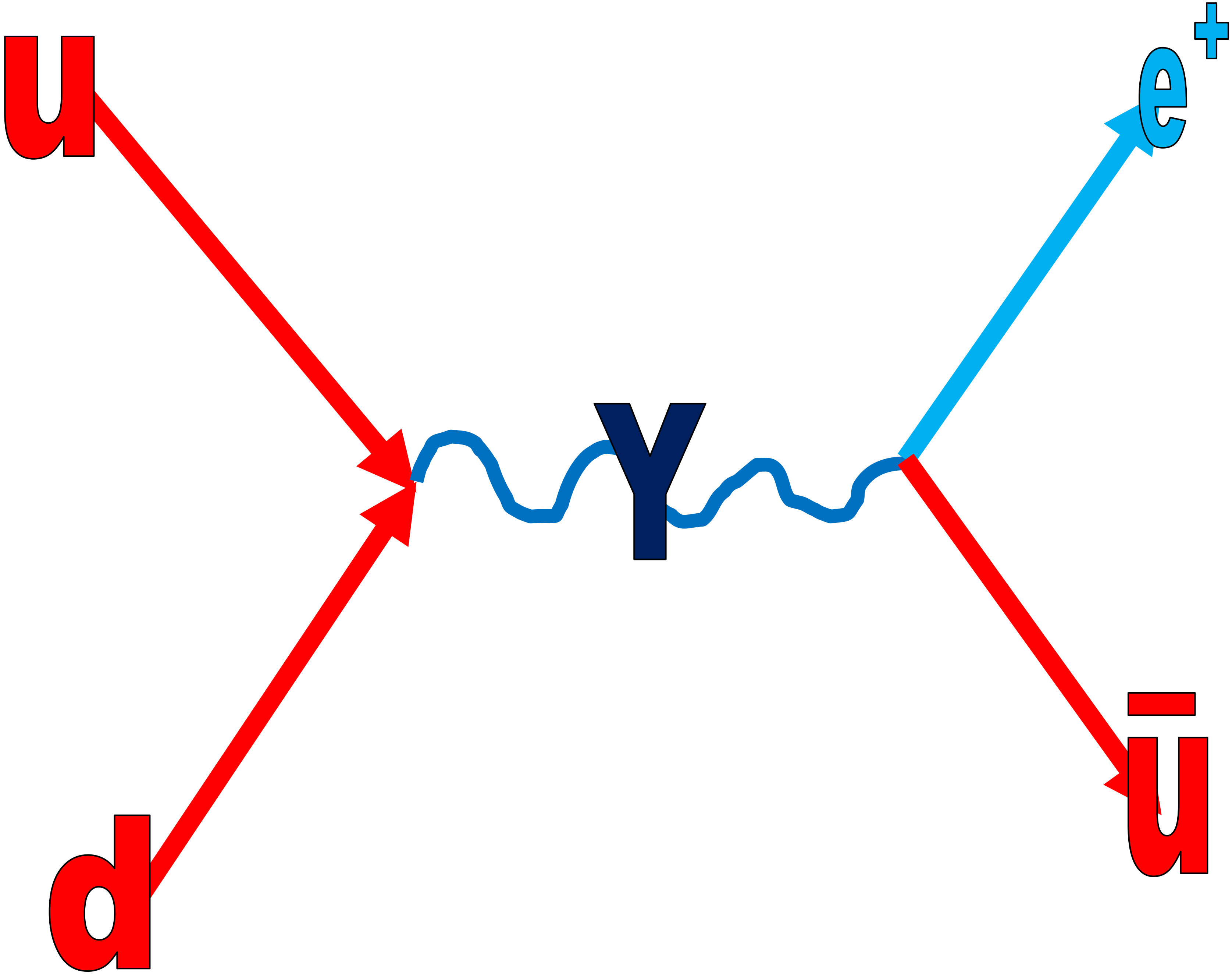
superheavy color triplet gauge bosons

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

(3,2) + (3*,2)

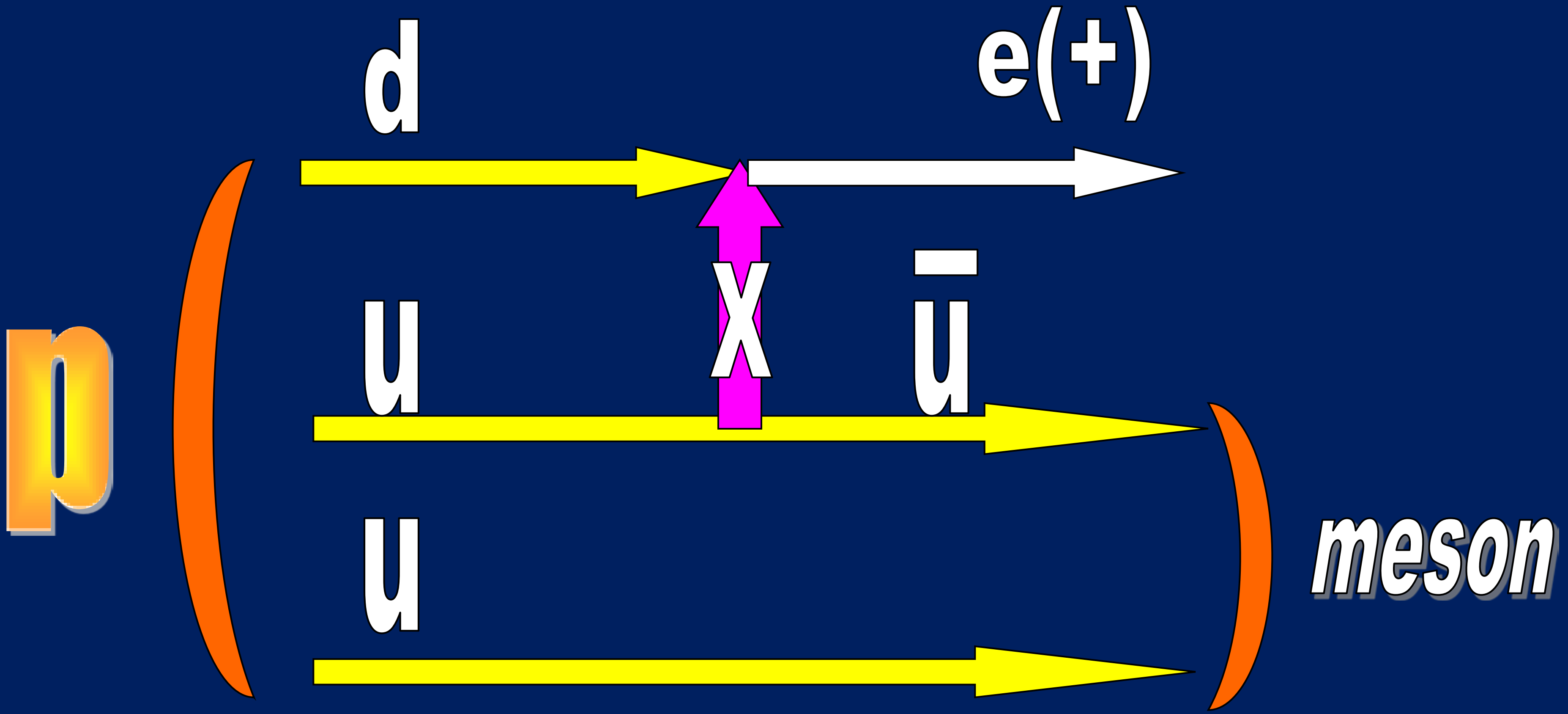


u
u
X
e+
d̄


u
d
γ
e+
ū

proton lifetime:

$$\tau(p) \approx \frac{M_{GUT}^{\;4}}{\alpha^{2}_{GUT} \cdot m_p^{\;5}}$$


d
e(+)
p
u
X
ū
u
meson

prominent possible decay:

$$p \Rightarrow \pi^0 e^+$$

(B-L) conserved!

expected lifetime:

10^{31} years

$e.g.:$

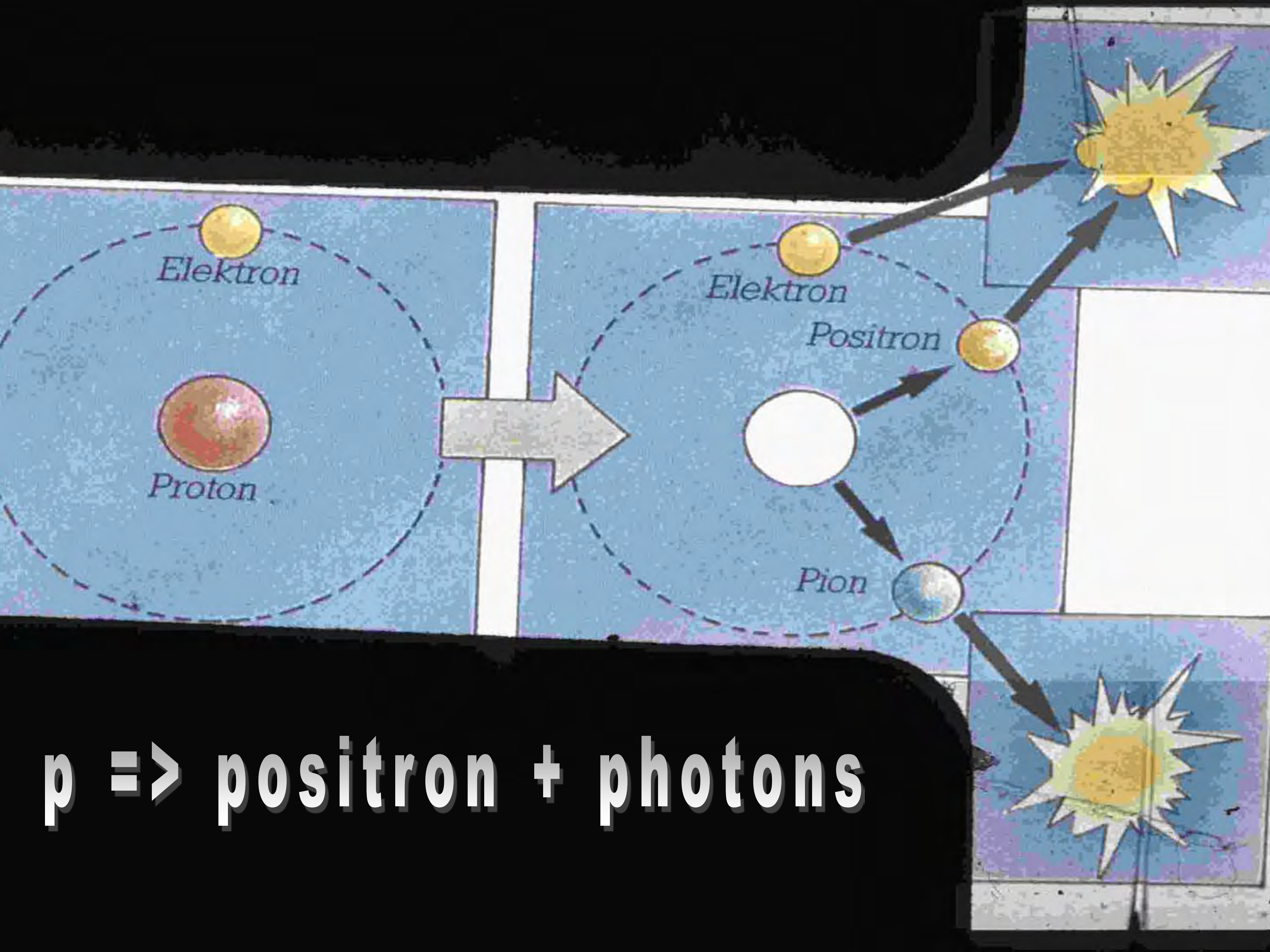
$$p = (uud) \Rightarrow e^{+}\pi^{0}$$

$$B = 1, L = 0 \Rightarrow B = 0, L = -1$$

B and L not conserved

but:

(B-L) conserved in SU(5) theory



p => positron + photons

$$p \Rightarrow \pi^0 e^+$$

exp. limit:

$> 10^{33}$ years

problem for SU(5)

Proton decay in SO(10):

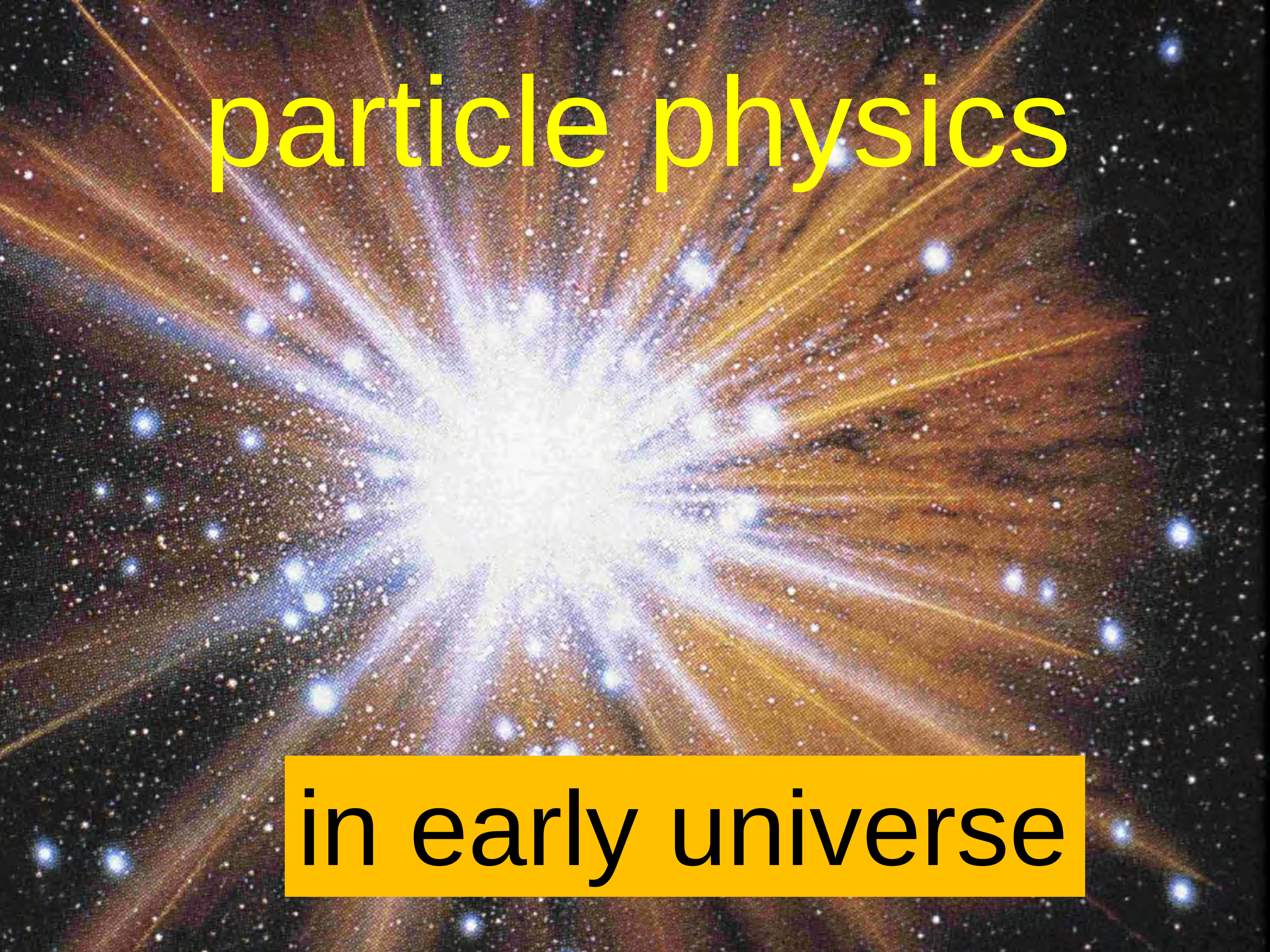
like in SU(5), but proton lifetime depends on unification energy, which is larger

$$\tau_p \approx 10^{34}\_years$$

Problem:

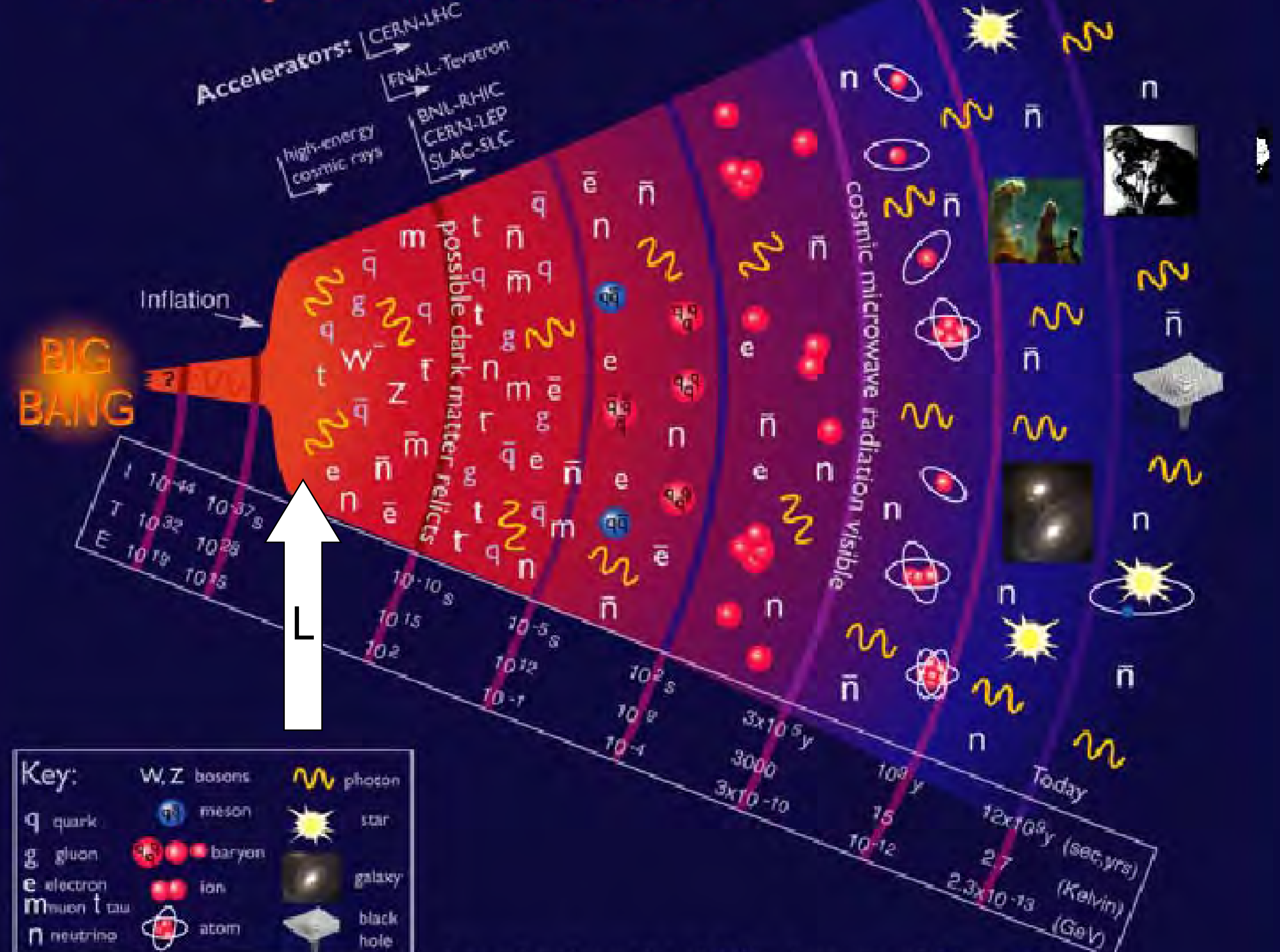
If proton has a lifetime larger than 10^34 years, the decay cannot be detected – neutrino background from cosmic rays

particle physics



in early universe

History of the Universe

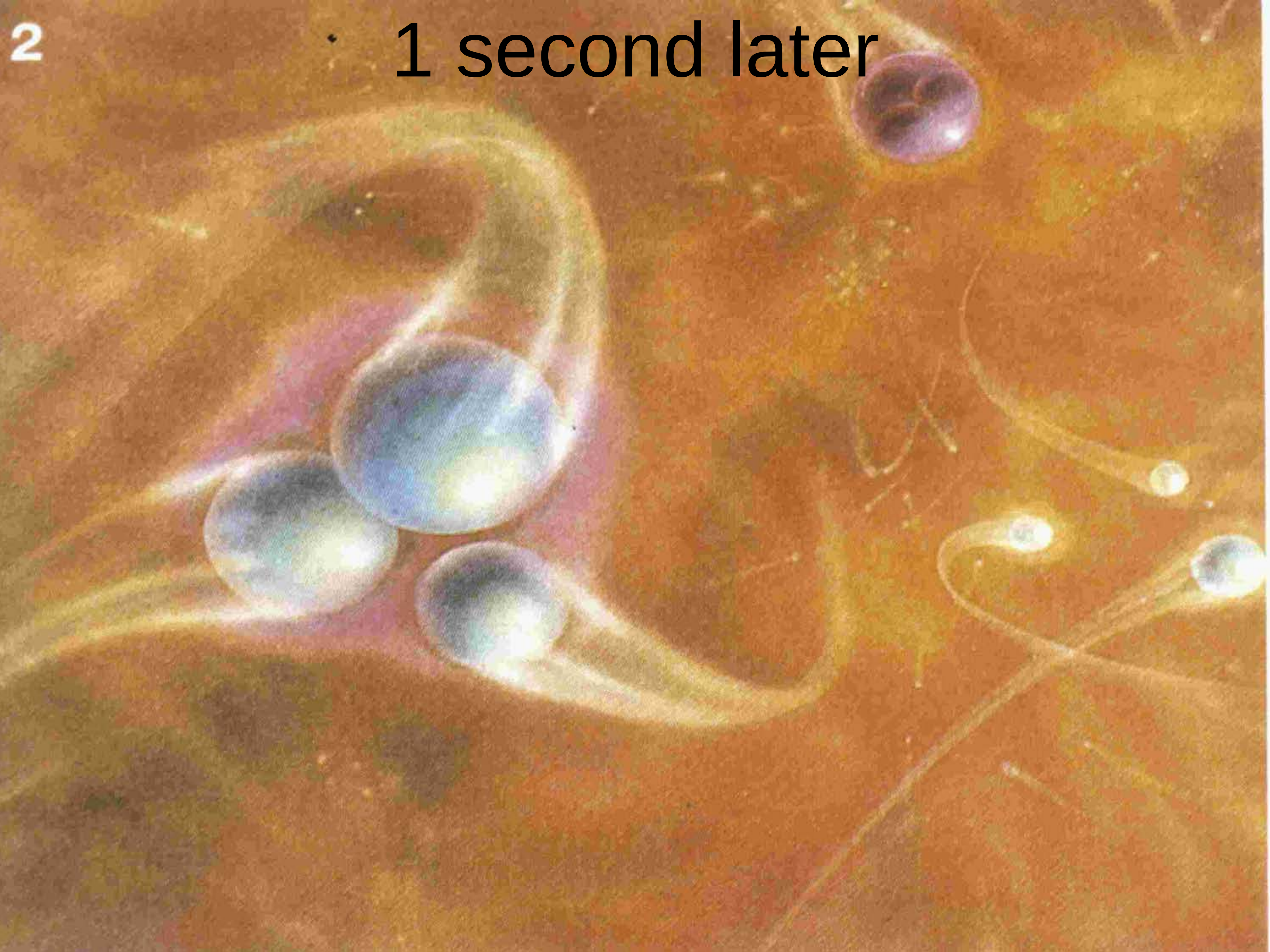


Particle Data Group, LBNL, © 2000. Supported by DOE and NSF quarks > antiquarks

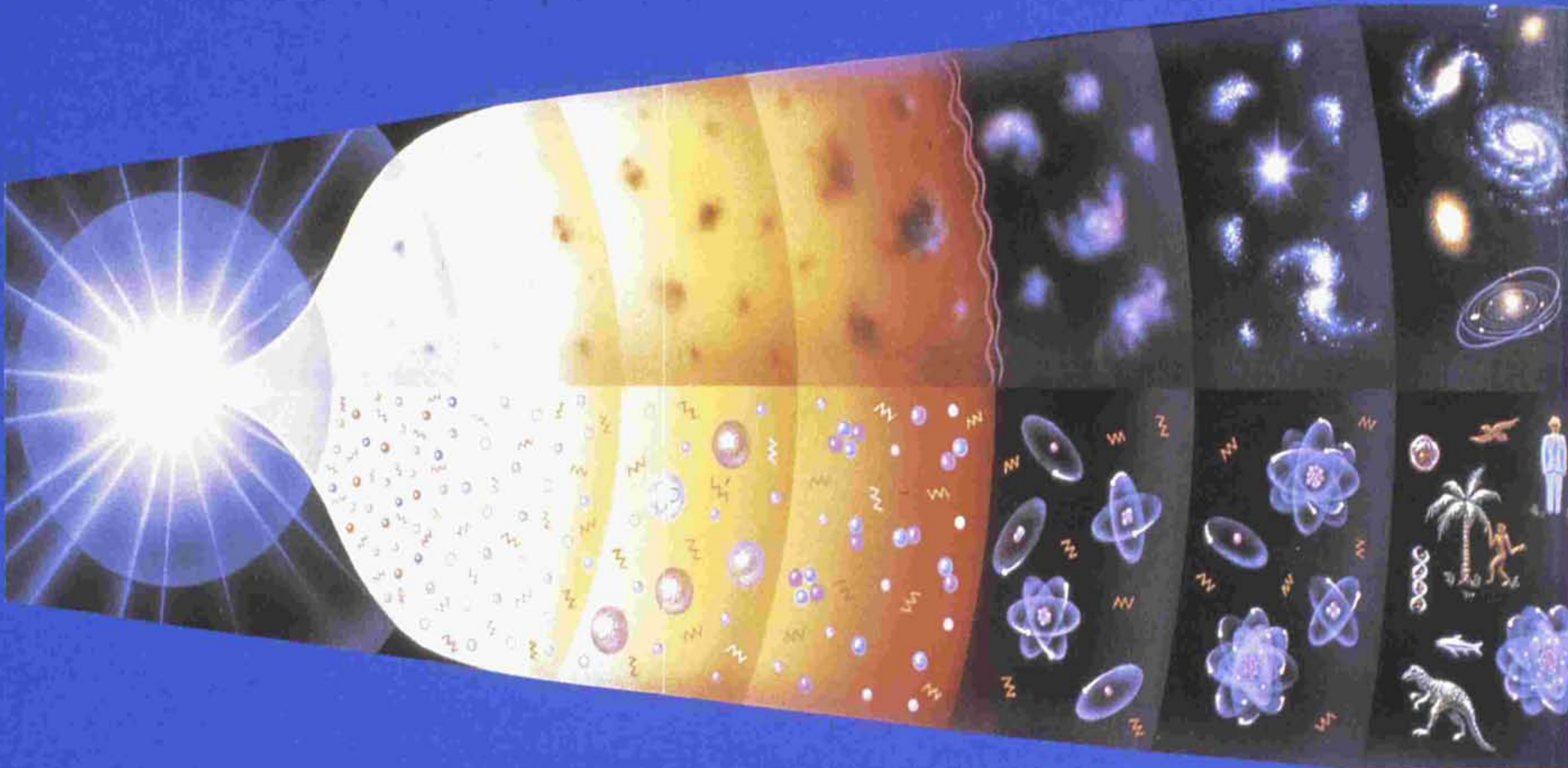
10 billions +1 10 billions

CP -violation

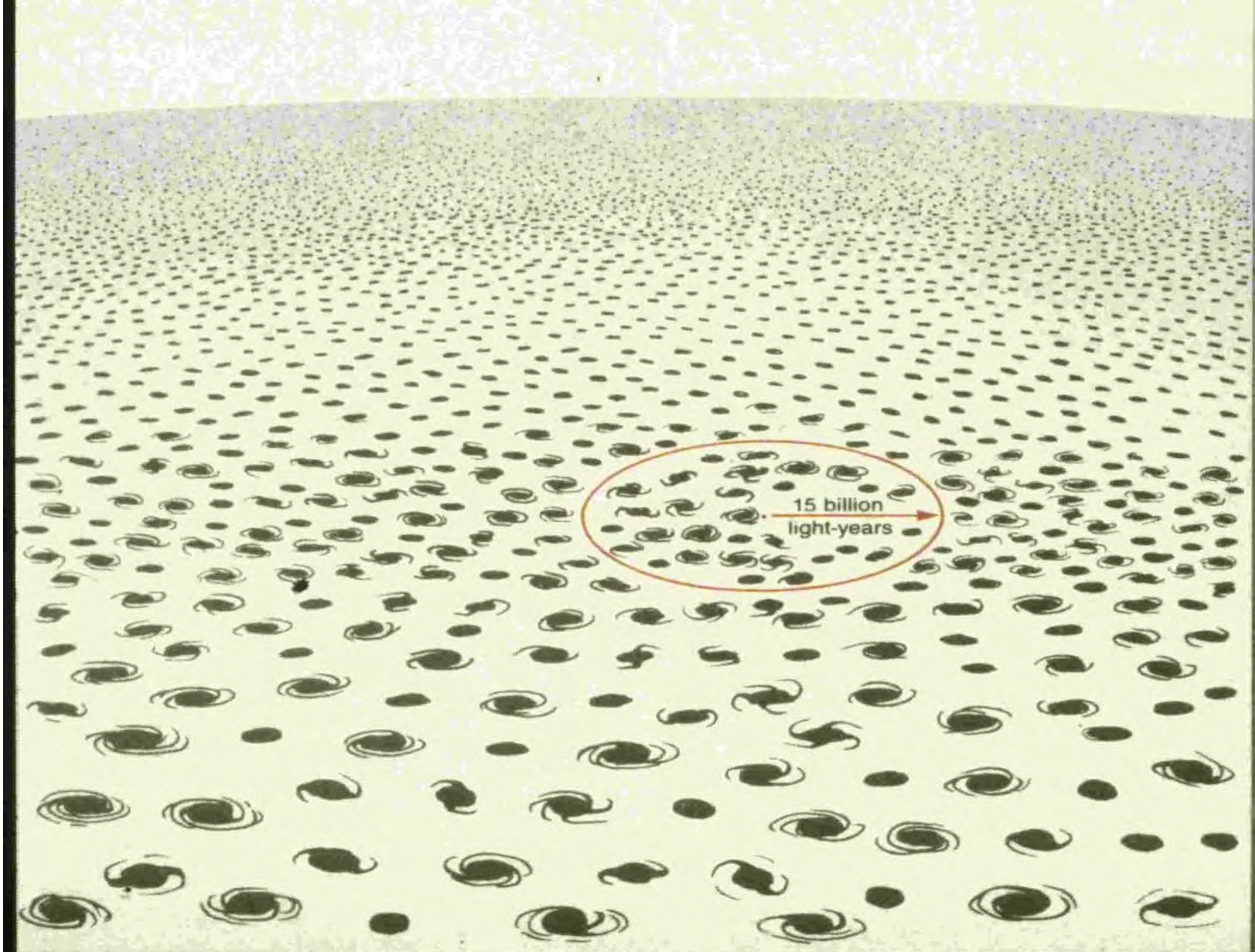
1 second later

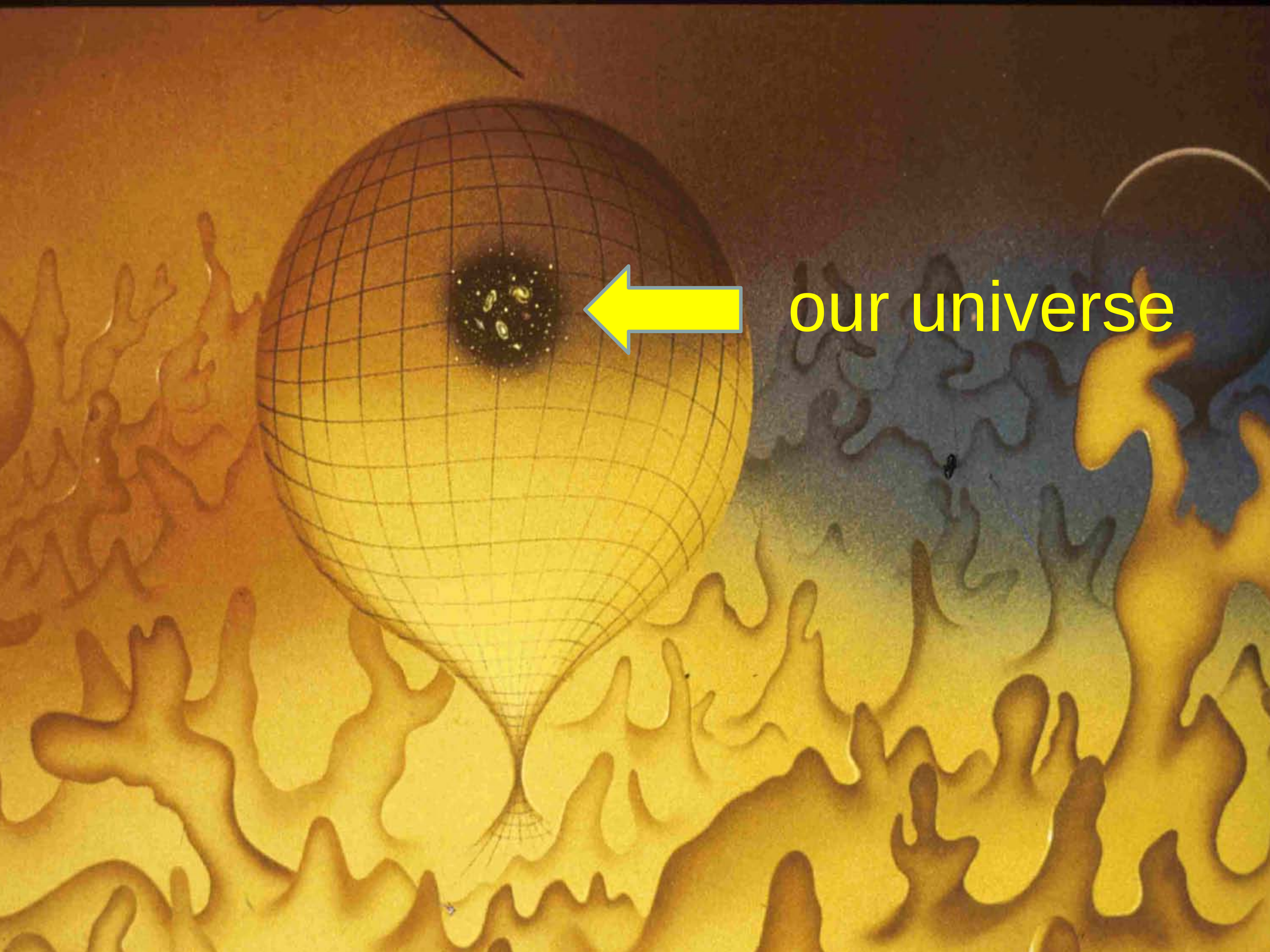


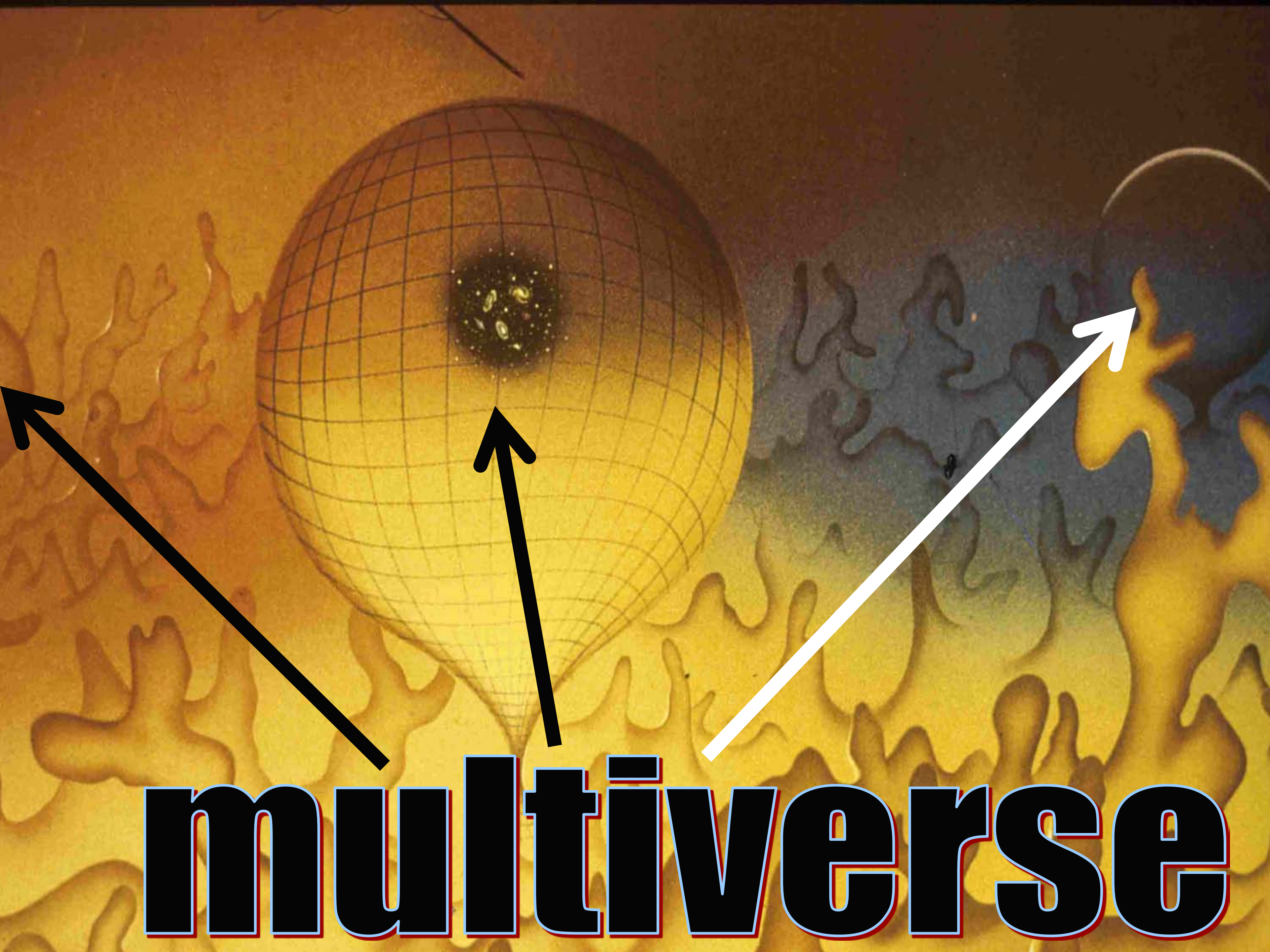
1 minute after bang He: 24%



inflation


15 billion
light-years


our universe


multiverse